

#### **FACULTY OF HEALTH AND APPLIED SCIENCES**

# **DEPARTMENT OF MATHEMATICS AND STATISTICS**

QUALIFICATION: Bachelor of Science in Applied Mathematics and Statistics					
QUALIFICATION CODE: 07BAMS LEVEL: 7					
COURSE CODE: MMO702S	COURSE NAME: MATHEMATICAL MODELLING 2				
SESSION: JANUARY 2019	PAPER: THEORY				
DURATION: 3 HOURS	MARKS: 100				

SECOND OPPORTUNITY EXAMINATION QUESTION PAPER					
EXAMINER	PROF. S. A. REJU				
MODERATOR:	PROF. O. D. MAKINDE				

	INSTRUCTIONS					
<ol> <li>Attempt ALL the questions.</li> </ol>						
2. All written work must be done in blue or black ink and sketch						
	be done in pencil.					
3.	Use of COMMA is not allowed as a DECIMAL POINT.					

# **PERMISSIBLE MATERIALS**

1. Non-programmable calculator without a cover.

THIS QUESTION PAPER CONSISTS OF 3 PAGES (including this front page)

## QUESTION 1 [20 MARKS]

(a) Describe Queuing modelling and hence state three basic performance measures of a queuing system. (7 Marks)

Consider a group flight schedule of a team of students to be airlifted to an excursion site by five aircrafts landing at an air field having two airport luggage service counters. The interarrival time and disembarking duration are as follows (in minutes).

Aircrafts	Flight 1	Flight 2	Flight 3	Flight 4	Flight 5	
Random Inter-Landing Times	20	30	15	120	25	
Disembarking Duration	55	45	60	75	80	

- (b) Construct a simulation table for the queuing model, showing all essential parameters.

  (9 Marks)
- (c) From your simulation table, determine the following performance measures:

(1 Mark each)

- (i) Mean wait time
- (ii) Mean disembarking time
- (iii) Mean time spent at the airfield
- (iv) Length of the longest queue

### QUESTION 2 [30 MARKS]

(a) Suppose we want to fit a polynomial of degree m of the following form:

$$y = f(x) = a_{m+1}x^m + a_mx^{m-1} + a_{m-1}x^{m-2} + \dots + a_2x + a_1$$

where  $a_{m+1}$ ,  $a_m$ , ...,  $a_1$  are to be computed to give the best polynomial fit. Derive the system of normal equations for computing the regression coefficients of polynomial fit y on x. (8 Marks)

(b) Given the following table of values for x and y:

x	1.0	1.5	2.0	2.5	3.0	4.0	5.0	6.0	7.0	8.0
у	1.11	1.32	1.63	2.04	3.45	4.16	3.2	5.4	6.1	7.5

and the following polynomial as the best fit for the above data

$$y = a_4 x^4 + a_3 x^3 + a_2 x^2 + a_1 x$$

Obtain the normal equations for the regression coefficients (correct to the nearest significant number), without solving the system. (22 Marks)

## QUESTION 3 [30 MARKS]

- (a) Consider a furniture maker that realizes a net unit profit of \$25 per table and \$30 per bookshelf. Assume that he has up to 690 metres of wood to devote weekly to the project and up to 120 hours of labour. He estimates that it requires 20 metres of wood and 5 hours of labour to complete a table and 30 metres of wood and 4 hours of labour for a bookshelf. Moreover, he has signed contracts to deliver four tables and two bookshelves every week. Formulate and solve the model maximizing his profits for tables and bookshelves. (9 Marks)
- (b) Define post-optimality analysis and hence discuss the analysis for change in his profits on tables, showing all expressions to support your conclusion. (11 Marks)
- (c) Discuss the sensitivity analysis for increasing labour hour to 150 hours, showing all expressions to support your conclusion. (10 Marks)

### **QUESTION 4** [20 MARKS]

- (a) Assume that in 2001, the population of Namibia was estimated to be 1.8 million and that by 2011, the population had grown to 2.1 million. Estimate the population growth and predict the population in the year 2020, with a zero migration, using the Malthusian assumption. (7.5 Marks)
- (b) (i) Discuss the limitations of Malthusian population model and hence construct a logistic model version, providing appropriate illustrations in your discussion.
  - (ii) Assuming a maximum population of 5 million, obtain the logistic model estimate of Namibia in 2020 using the growth rate obtained in (a). (11.5 Marks)

**END OF EXAMINATION**